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Mr. Gompertz attempts to prove that I had used the correct but superfluous quantity (L_0) as distinct from unity. This he entirely fails to do, for he reproduces the identical figures which were given by me as obtained by the substitution of unity for L_0.

*On the various methods pursued in the Distribution of Surplus among the Assured in a Life Assurance Company; with a comparison of the relative merits of such methods.** *By* WM. POLLARD PATTISON, ESQ., *of the London and Provincial Law Assurance Society.*

[The Author of this Essay obtained for it the Prize offered by the Institute for an Essay on the subject.—ED. *A. M.*]

THERE is, probably, no department of an actuary's pursuits where there is such diversity of practice as that of which this paper treats. After the calculation of the premiums and the periodical valuations, the distribution of surplus is infinitely the most important duty that an actuary has to perform. The premium income of the various Life Offices now exceeds £8,000,000 a year, and probably not less than £1,000,000 is annually divided as surplus. Within the last 14 years, one Office alone has divided upwards of £1,200,000; and another, for many years past, has annually distributed as surplus more than £100,000. This will be sufficient to show the magnitude of the interests concerned; and yet, as will be seen in the sequel, these distributions are, in some cases, made in a most arbitrary manner—without reference to principles of justice and equity, and without the basis of accurate reasoning. After what has been written on this subject by Mr. Jellicoe and Professor De Morgan, it is not to be supposed that any actuary would defend the schemes here referred to as equitable; those who *do* follow them, content themselves with stating their advantages. In many cases, however, their conservatism arises from the difficulty of disturbing vested interests; and the maxim, "*Omnis innovatio plus novitate perturbat quam utilitate prodest,*" is very attractive to those in office, upon whom would fall the labour of carrying out a new system. Hence, in this, as in many other cases of old-established errors, all true reform must come from without. But the application of principles of reform is very difficult in the matter under consideration. It is a subject which can never become popular. The assuring public will never enter into the abstract merits of any particular scheme; but if the comparative numerical results of each one be clearly exhibited, the effect must be that the adoption of equitable methods would be forced upon the different Offices. Each would gradually lose the custom of any class of lives to which its method was known to be unfavourable, and the favoured classes would ultimately be drawing profits contributed only by themselves. In addition, therefore, to a statement of what I hold to be correct principles, and a description of the various methods in practice, I have appended a table of comparison, which, at a glance, will show the working of any particular scheme as compared with any other, in a manner that can be sufficiently understood even by persons who are not familiar with such matters.

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* By the title of this paper it will be perceived that the methods actually pursued are those which are the subject of investigation and comparison. I emphatically wish it to be understood that they are in no sense theoretical, but such as are adopted at the present time, and mostly, too, by Offices of high standing. The names have been carefully excluded, and everything that might lead to their identification.

The need for an equalization of the risks of death called into existence the plan of life assurance. The theory upon which this system depends, presupposes that the duration of life in a community can be accurately measured. Every year additional facts are acquired, and every year tends to confirm the soundness of the theory, though it modifies its application. The theory also presupposes that the rate of interest at which money will accumulate can be foreknown; a matter on which our predictions can never be more than approximately accurate. The uncertainty thus caused by modifications in the laws of mortality and the fluctuations in the rate of interest, apart from expenses of management, require that a marginal addition (technically termed a "loading") should be made to the estimate of the risk, sufficient to provide for the probable divergencies of the future from the experience of the past. Hence the premiums charged are greater than have been hitherto found sufficient for the risk; and this brings us to the consideration of the surplus, which may be traced to—

1. The loading, or margin for guaranteeing the engagements of the Company against the fluctuations of mortality and the rate of interest, for providing for the expenses of management, and for forming a bonus fund:
2. Interest—a higher rate being realized than was assumed: and,
3. Miscellaneous sources, lapses, surrenders, non-profit policies, office age over real age, fines.

The premiums charged to the public consist of two parts—(1), that which simply provides for the risk; and (2), the loading, the whole or a portion of which is at intervals found to be surplus, and accordingly distributable among the assured. In respect of the former portion of the premiums, the assured form a community of assurance; and in respect of the latter portion, a community of profit. The only principle of the division of the surplus which has been advocated, is, that each of the assured should receive in proportion to his contribution to the profit fund. I have already pointed out that, as regards one portion of the premium, the assured may be considered as a community of profit—the criterion of all partnership; and the principle just laid down is that which guides all partnership contracts, whether the contribution to profits be property or labour. Now, the error of certain systems of distribution is owing, not to the denial of this principle, which no one has, as far as I know, ever attempted to controvert, but to the almost wilful refusal to connect the sources of surplus with the contributions of the members, without which connexion the principle of proportional distribution cannot be applied.

Let p = premium charged;
p' = premium for the risk, or true premium;
ϕ = loading, or $p-p'$;
a^n = amount of an annuity of £1 for n years;
s = total amount of divisible surplus; and
Σ be the symbol of summation.

The valuation being assumed to have been made by a true table of mortality and at a true rate of interest—and if not so based, the real surplus would obviously not be known at all—the principle would be correctly applied by the formula $\frac{s}{\Sigma\phi a^n}\phi a^n$,* to represent the share of the assured in the divisible surplus. These assumptions apply to the valuation alone; the table on which the premiums are based has nothing to do with the question. When p is not more than p', then there would be no bonus, as should be the case. The application of the principle by the above formula includes the assumption that the expenses of management are paid out of the miscellaneous profits which have not been contributed to by those who participate in the surplus; or, if these sources of profit will not pay the whole expenses, then it assumes that the management should be borne *pro ratâ* to the loading. I shall hereafter refer to this point, and, for the present, it will be convenient to exclude it from the general question which is the subject of this paper. Since the formula depends on the rigid fulfilment of the above conditions, it is necessary to inquire how far they can be obtained.

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* *Vide* Mr. Jellicoe's papers in the *Assurance Magazine*, vols. ii. and iii. I wish this to be considered as a general reference, applying to other portions of this Essay.

1*st. A true table of mortality.*—The English Life Table of the general population of the country may be safely considered as representing the mortality of the majority of Offices—certainly all with a large proportion of country business; and Mr. Morgan's Equitable Tables, as representing the mortality of the better class of assured lives. The valuation surplus would probably be derived very largely at the first and second periodical divisions, and afterwards, in a less degree, on account of suspended mortality, arising from recent selection of the lives; but this portion of the surplus ought not to be divided, inasmuch as the subsequent mortality cannot safely be assumed to follow the average. To ascertain the divisible surplus, deduction from the valuation surplus ought to be made for suspended mortality. Hence, then, no profits can be said to be made from that source; should they appear, inappropriate tables have been employed, and all certainty of profit gone. For these considerations, I would disregard this source of profit from the question of the true mode of distributing profits.

2nd Condition. A true rate of interest.—I would almost venture to say, that this cannot be fulfilled. While, on the one hand, we find 3½ and 3¾ realised by two Offices of the highest standing, so, on the other, there are Offices making 4¼ and 4½, and in one first-class Office very nearly 5 per cent. is made, with the life funds, through preference of investments being granted to that portion of a large accumulation of money. Four per cent. has in some valuations been assumed as the true rate. Now, it is to be observed, that with the accumulation of funds the difficulty of maintaining the rate of interest increases; and further, it depends upon good management. An Office may have now a skilful financier as its actuary; he would, doubtless, advocate a 4 per cent. valuation. But should the management change, and a few men, with prejudices for and against particular securities, become members of the Board, 3½ per cent. might be the rate of interest—and all this apart from the fluctuations in the value of money. Supposing £1,000,000 to be the liability by a 4 per cent. valuation, the change in the base to 3 per cent. would diminish the divisible surplus by upwards of £100,000; and this might cause considerable alarm among the Office connexions. I would, therefore, adhere to a lower rate of interest—3, or, at most, 3½ per cent.—and introduce, as a further element in the ratio of division, the profit from this source. From the total divisible surplus, the profit so derived would be deducted, and would be composed of two parts—

1. To be divided among all that have paid premiums within the period; and,
2. to be divided among those who participate in the previous division.

It is clear that, if at the last valuation 3 per cent. were assumed and 4 per cent. have since been realised, the sum then reserved was more than sufficient, and the difference ought to be returned, as far as practicable, to those who would then have participated in a larger amount of surplus. This element in the distribution might be easily introduced where the premiums are based on the true table of mortality, and where the loading is wholly a percentage on the premiums or a percentage with a small constant for expenses. The following methods of distribution all omit a proportion of profit from interest to those who participated in a former division; and all but the first, the second (where the loading is a percentage), and the third (where the loading is a percentage and a constant), with respect to the increased interest obtained within the last period. In giving my opinion, therefore, that any one mode is equitable, it will be always subject to modification in the case of the interest obtained being more than the assumed rate.

The following are the principal methods at present pursued:—

1. In proportion to premiums paid since the last division, accumulated at compound interest to date of valuation.
2. In proportion to the loading—that is, the difference between true and Office premiums—accumulated at compound interest between the intervals of division.
3. In proportion to the difference between the pure and Office premiums, less a constant for expenses, accumulated at compound interest between the intervals of division. (This is adopted, with some modification.)
4. In proportion to the difference between the premiums paid, accumulated with interest, and the respective values of the policies.
5. In proportion to the values of the policies.
6. A reversionary bonus in proportion to the premiums paid, without interest, always reckoning from the commencement of the policy.
7. In augmentation of the sums assured by a percentage per annum on the sums assured and the bonuses already

declared, the number of years being reckoned back to the last distribution of profits only.

8. In augmentation of the sums assured by a percentage per annum from the date of the policy.
9. In reduction of the annual premiums by an uniform percentage to all members after payment of a fixed number of full premiums.
10. In proportion to the amounts assured under the policies which have become claims within the year.
11. Amongst those of the assured only whose premiums, accumulated with interest, exceed the amounts assured under their policies.

1st method.—In proportion to premiums paid since the last division, accumulated at compound interest to date of valuation.

This is the most general mode of distribution, and if the premium is formed from the true premium for the risk, and loaded by a percentage, it is equitable; and, which is a great recommendation, is a practically good mode. By "practically good," I mean that it produces that proportion between the bonuses at different ages which will give most general satisfaction among the assured.

Using the same notation, the working of this will be shown by the formula $\frac{s}{\Sigma pa^n}pa^n$; and if ϕ be a percentage on p', and, as before, $p=p'+\phi$, then $p:\Sigma p::\phi:\Sigma\phi$. Hence this formula will give the same results as $\frac{s}{\Sigma\phi a^n}\phi a^n$.

But though this method of distribution, under the assumption of a percentage loading, is equitable, such a loading is very rare in practice; and according as the loading diverges from this formation, so does the method depart from what is just. I have known this method applied where Northampton premiums were charged. Its effect with such premiums will be readily seen from the following:—

Age.	Northampton.	English Life Table.	Δ.
	Premium per Cent. per Annum.	Premium per Cent. per Annum.	
	£ s. d.	£ s. d.	s. d.
25	2 8 1	1 15 9	12 4
55	5 6 4	4 16 9	9 7

If the English Life Table be held as the true rate, the respective shares in the surplus at ages 25 and 55 at entry would be as 24 : 53, or about 4 : 9, though age 25 contributes annually to the surplus 2*s.* 9*d.* per cent. more than age 55. Were commission taken into account, the unfairness of the method applied to such premiums would be still more manifest.

If Mr. Morgan's Equitable be taken as the true rate, we have—

Age.	Northampton.	Equitable.	Δ.
	£ *s.* *d.*	£ *s.* *d.*	*s.* *d.*
25	2 8 1	1 17 2	10 11
55	5 6 4	4 14 7	11 9

that is to say, the ratios of division would be as 4 : 9, though the contributions were nearly equal, viz., 10*s.* 11*d.* and 11*s.* 9*d.* The Northampton rates exhibit the unfairness in its highest degree, but there are many others which to a less extent are unfair. Before passing on to the next method, I would notice that the ratio of the premiums paid would be a fair ratio for dividing that portion of the profit which has arisen from excess of interest derived on the premiums paid *within the period since the last division.* The profit from interest on the reserve for liabilities made at the former division would have to be divided among those only who then participated. I have remarked upon the expediency of this method of distribution, and upon this point it is to be further noticed that the ratio of division (premiums accumulated within the intervals of division) is, after the first division, a constant. The successive bonuses being equal, the share of each in cash will also be equal, and the reversionary bonuses with advancing years will continuously diminish. Now the reversionary bonus is the measure among the public, and the reason for the diminution is not understood. This result has, of course, nothing to do with the equity of the method, whatever it may have with its expediency. If the view of the public with respect to increasing bonuses were incompatible with equitable distribution, it would be necessary to dispel the notion, difficult as that might be. This, however, is not necessary. Mr. Sprague has pointed out that the cash bonus may be applied as an increasing assurance; and there is the plan pursued by some Offices of converting the cash into reversion by the table of single premiums given in the prospectus for participating policies, and then giving bonus on bonus in future divisions. When the Office is "Mutual," no objection can be urged on the ground of principle; but when the Office is "Mixed," the effect is to give to the shareholders a portion of a former division, although they have already received their full share. This injustice is increased by the conversion of the cash bonus into reversion by the table above quoted, which is very rarely used when the assured have the option. The rates include a charge for Office expenses and commission; so that when the assured have a cash bonus, a portion of the two-thirds or four-fifths, as the case may be, to which they alone are entitled, that portion is taxed with a charge for the management of it, with commission which is never paid, and with a contribution one-third or one-fifth of which will go to the shareholders, though such contribution be a portion of the two-thirds or four-fifths of profits, to which they have no title whatever. If this system be advisable, deduction should at least be made of that portion of the loading, about one-half, which provides for expenses and commission.

2nd method.—In proportion to the loading—that is, the difference between true and Office premiums—accumulated at compound interest between the intervals of division.

It has before been stated, that any profit that may arise will be from ϕ; and if this be the measure of each one's profit, this method satisfies the only principle that has been laid down as equitable.

The application of this method is extremely easy, various as may be the premiums payable under the policies. The existence of twenty different rates of premiums would not be the least obstacle to its application.

*3rd method.—In proportion to the difference between the pure and the Office premiums, less a constant for expenses, accumulated at compound interest between the intervals of division.**

This is a modification of No. 2 method. Nos. 1 and 2 both assume that the expenses of management should be paid out of the general surplus fund, so that each one's share of expenses would be in proportion to his contribution to surplus. Now, if this be a constant, his share of the expenses is also a constant. This I hold to be correct. One £100 policy costs (disregarding commission, which is a percentage) as much per annum as another, whether the premium be £5 or £2. The charge for this purpose should therefore be the same at all ages. If this principle be admitted, the second method is erroneous to this slight extent, and the third is perfectly equitable. This constant need not be sufficient for the full amount of expenses, as the miscellaneous profits may be justly set off to diminish these. Could they be found sufficient, the second method would be equitable, and, as no constant would then have to be deducted, the third method would coincide with it.

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* The working of this is, I believe, not exactly as described. The accumulation is made by a table of temporary annuities; and the former bonuses are taken into account in determining the ratios for subsequent divisions.

4th method.—In proportion to the difference between the premiums paid, accumulated with interest, and the respective values of the policies.

The application of this at the first division of profits is very simple; and, supposing x to represent the ages at entry, the formula for working would be $\frac{s}{\Sigma(p_x a^n - l_x)} \times (p_x a^n - l_x)$ for the share of each policy. At the second and subsequent divisions, it becomes more complicated. All the policies that participated in the former division are, for the purpose of the second division, considered as new policies effected at the first division. Thus, if a man assure at age 25, and is 32 at the first division, the accumulated premiums paid, less the value of a policy 25 at entry, after seven years, will express the ratio of his share in the surplus at the first division. If the second division be seven years after the first, seven years accumulations of premiums payable at the age of 32—his age when the first division took place—less the value of a policy on the life of 32, after seven years, will express the ratio of his share at the second division. There are modifications of this in some instances, as regards the bonus. The bonuses declared at previous divisions are added to the amount assured, and the premiums are formed upon the augmented amount assured. Thus, if in the example before given £100 were the bonus at the first division on a policy for £1,000, the ratio for the second division would be the accumulated premiums on £1,100 at age 32, less the value of a policy of £1,100 seven years after inception at that age.

The formulæ for the working of this method at the second division, without and with the modifications as regards bonus, would be—

$$\frac{s}{\Sigma(p_{x+7}a^7 - {}^7l_{x+7})} \times (p_{x+7}a^7 - {}^7l_{x+7})$$

and

$$\frac{s}{\Sigma\{p_{x+7}(\mathrm{M}+\mathrm{B})a^7 - {}^7l_{x+7}(\mathrm{M}+\mathrm{B})\}} \times \{(p_{x+7}(\mathrm{M}+\mathrm{B})a^7 - {}^7l_{x+7}(\mathrm{M}+\mathrm{B})\},$$

M being the amount originally assured, B the bonus, and in the second formula p and l are the premiums and values per £1 assured. The first of these formulæ is to be preferred, as the working of it is more simple, and it does not further augment the bonuses on policies of long standing, which are otherwise too large.

The principle on which this method seems to have been founded is this, that the accumulation of all premiums, less the value of all the policies, would represent the gross surplus, if there were no expenses and no claims, and the contribution of each to that gross surplus would be the accumulation of the premiums he had paid, less the value of his policy. The divisible surplus being the amount of such imaginary surplus, less the claims and expenses, the share of each in the divisible should be represented by the ratios in the imaginary surplus. This notion includes the fallacy, that if a man aged 55 lives over a period, say of five years, he has contributed more profit than one aged 25 living five years, apart from the question of the loading of the premium.

Now, it has been already shown that this is contrary to the very essence of life assurance. In a true table of mortality, no profit is derived from long life, except as regards that portion of the premium which expressly provides for profits and expenses. If this distribution between the two parts of the premium be not regarded, all consistency in our deductions will disappear.

Referring again to the formula $\frac{s}{\Sigma(pa^n-l)}pa^n-l$, if we substitute for p its equivalents $p'+\phi$, and $\therefore\Sigma(p'+\phi)a^n$ for Σpa^n and for Σl, the net liability on all the policies $\Sigma(p'a^n-c)$; for it is clear that if p' represents the true premiums, the amount which should be in hand at any time would be the sum of the amounts of every p' or $\Sigma p'a^n$ less the total amount in claims, or Σc; so also for l we may substitute $p'a^n-c$ (c being p' contribution to claims), we then have

$$\frac{s}{\Sigma(p'+\phi)a^n-\Sigma(p'a^n-c)}\times\{(p'+\phi)a^n-(p'a^n-c)\},$$

or

$$\frac{s}{\Sigma(\phi a^n+c)}\phi a^n+c;$$

that is to say, the profit is allotted in the proportion of the contributions to profits and the payment of claims, which has been shown to be contrary to principles of equity, and to include an assumption opposed to the fundamental principles of life assurance.

5*th method.—In proportion to the values of the policies.*

By this method the surplus is divided in proportion to the liability under each policy, and the working of it is expressed by the formula $\frac{s}{\Sigma.l}.l$.

How inequitably this system works will be seen by the table of comparison appended, and this is with a well-graduated premium upon the same base as the valuation. If applied in an Office where different rates are payable—some with a large margin, others with a small or no margin—those paying a large marginal addition will have the same bonus as those paying a small one (the ages of the lives and the dates of policies being the same), and even a less bonus than a policy with a premium inadequate for the risk. It is, however, to be observed, that if all the participating policies in an Office paid according to the same scale of premiums, and this were formed on a true base, with a percentage and small constant for loading, this method would provide to the new policies compensation in future years, since the value of every policy, whether effected on a young or old life, would, if the life attained the extreme age in the table, range from 1 to 99 per cent. The ratios of division, therefore, range between the same limits for all policies.

6th method.—A reversionary bonus in proportion to the premiums paid, without interest, always reckoning from the commencement of the policy.

The clause in the deed wherein this scheme was devised runs as follows:—

> "The dividend to each member shall bear the same proportion to the total sum to be divided as the total premium received of the existing members insured for life, which sum allotted to each individual shall be added to the sums originally insured."

The Office, I believe, interprets this clause to mean, that the reversionary bonus is to be in proportion to the premiums paid, not the cash bonus, as it has been reasonably considered to mean, on the ground that the sum to be divided being cash, the fourth term of the proportion must be like, and also cash. There is no trace of principle in this. The profits are given to the long livers, and in greater proportion than by any other scheme.

7th method.—In augmentation of the sums assured by a percentage per annum on the sums assured and the bonuses already declared, the number of years being reckoned back to the last distribution of profits only.

This is unscientific; but, by a graduated loading, this plan could be made nearly equitable. It would not, however, admit of adjustment, should subsequent experience render it desirable to modify the table of mortality.

It will be seen in the table of comparison, that according to the loading there made, it favours the young lives. How far the plan of giving bonus on bonus is inequitable, depends on the way in which the cash surplus is converted into a reversionary sum, and on the Office being " Mutual " or " Mixed."

8th method.—In augmentation of the sums assured by a percentage per annum from the date of the policy.

The origin of this is well known to have been the discovery that the experimental premiums charged to the early assured were greatly in excess of the risk, and this was devised to restore the equality. It could never be adopted again, and nothing but a great reputation for wealth could sustain an Office against the inexpediency of the method. The application of the principle varies in different Offices. The Office in which this method originated converts the cash bonus into reversion, and then divides this according to the above principle. Another Office, however, applies the principle of distribution to the cash surplus, and, though it is a *cash* share of profits, it is not payable until death. The effect of this in a proprietary Office, giving only, say two-thirds, of the profits, may be readily conceived. Assuming the surplus to be £300,000, the shareholders would get £100,000, and the policyholders £200,000; but since they would only come into this on the falling in of the policies, the present value would not be much more than £100,000. The surplus at the next division would consequently be increased by the remaining £100,000 accumulated, with interest. But here again the shareholders would get their one-third share, and of the policyholders' £100,000 accumulated as part of the general surplus.

The above-described methods are all that are included in the table of comparison before referred to. I append this table, which contains the different methods, arranged in the order I have described them.

The policies are all assumed to be effected in the commencing month of the quinquennial period in which they first participate. They are all for £1,000; and the ages at entry are 25, 40, and 55. The premiums are according to the 3 per cent. English Life Tables (published in the *Registrar-General's Report for* 1849), with loading of 10 per cent., and a constant of 5*s*. per cent. This loading has been adopted to obtain an average premium, and not from any reasons for that specific loading. The calculations have been made according to the same tables.

Table of Comparison.

Calculations based on the English Life Table (Males), 3 per Cent.

Age at Entry.	Premium payable.	Age at Valuation.	No. 1.		No. 2.		No. 3.		No. 4 (i).		No. 4 (ii).		No. 5.		No. 6.		No. 7.		No. 8 (i).		No. 8 (ii).	
			Cash.	Reversion.	Cash.	Reversion.	Cash.	Reversion.	Cash.	Reversion.	Cash.	Reversion.	Cash.	Reversion.	Cash.	Reversion.	Cash.	Reversion.	Cash.	Reversion.	Cash.	Reversion.
25	22·16	30	20·	48·6	25·4	61·5	19·	46·	21·4	51·9	21·4	51·9	18·3	44·4	14·2	34·5	25·5	61·8	25·5	61·8	..	33·3
40	32·70	45	29·5	56·	31·	58·7	29·4	55·6	29·2	55·3	29·2	55·3	30·	56·9	26·8	51·	32·6	61·8	32·6	61·8	..	33·3
55	55·80	60	50·5	74·4	43·6	64·3	51·6	76·	49·4	72·8	49·4	72·8	51·7	76·2	59·	87·	41·9	61·8	41·9	61·8	..	33·3
First Period			£100·		£100·		£100·		£100·		£100·		£100·		£100·		£100·		£100·		.. £	100·
25	22·16	35	20·	44·8	25·4	56·9	19·	42·5	20·2	45·2	20·7	46·3	25·8	57·9	19·5	43·6	27·3	61·1	34·9	78·2	..	44·4
40	32·70	50	29·5	51·4	31·	54·	29·4	51·2	32·4	56·4	30·	52·3	41·4	72·	36·9	64·4	35·1	61·1	45·	78·2	..	44·4
55	55·80	65	50·5	69·3	43·6	59·8	51·6	70·9	62·4	85·4	60·5	83·	66·7	91·4	80·1	110·	44·5	61·1	57·0	78·2	..	44·4
25	22·16	30	20·	48·6	25·4	61·5	19·	46·	18·3	44·4	19·	46·	12·1	29·6	9·	21·8	23·7	57·6	16·	39·1	..	22·2
40	32·70	45	29·5	56·	31·	58·7	29·4	55·6	24·8	47·1	26·	49·3	19·8	37·4	17·	32·2	30·4	57·6	20·6	39·1	..	22·2
55	55·80	60	50·5	74·4	43·6	64·3	51·6	76·	41·9	62·	43·8	64·6	34·2	50·4	37·5	55·3	39·	57·6	26·5	39·1	..	22·2
Second Period			£200·		£200·		£200·		£200·		£200·		£200·		£200·		£200·		£200·		.. £	200·
25	22·16	40	20·	41·2	25·4	52·1	19·	39·1	20·6	42·5	20·1	41·4	30·3	62·4	22·9	47·4	29·3	60·3	40·4	83·4	..	50·
40	32·70	55	29·5	47·2	31·	49·6	29·4	47·	32·1	51·3	31·2	50·	48·6	77·8	43·	69·	37·7	60·3	52·3	83·4	..	50·
55	55·80	70	50·5	65·	43·6	56·	51·6	66·4	78·6	101·3	73·	94·	72·4	93·3	91·5	117·9	46·8	60·3	64·9	83·4	..	50·
25	22·16	35	20·	44·8	25·4	56·9	19·	42·5	17·7	39·6	18·2	40·6	19·1	42·9	14·1	31·3	25·4	56·8	24·8	55·6	..	33·3
40	32·70	50	29·5	51·4	31·	54·	29·4	51·2	25·7	44·8	26·4	46·	30·6	53·1	26·3	46·	32·6	56·8	32·	55·6	..	33·3
55	55·80	65	50·5	69·3	43·6	59·8	51·6	70·9	52·6	72·3	53·	72·7	49·8	68·2	57·2	78·8	41·4	56·8	40·6	55·6	..	33·3
25	22·16	30	20·	48·6	25·4	61·5	19·	46·	15·6	37·8	16·8	40·7	9·	21·9	6·4	15·6	22·1	53·6	11·5	27·8	..	16·6
40	32·70	45	29·5	56·	31·	58·7	29·4	55·6	21·2	40·2	22·8	43·	14·7	28·	12·1	23·	28·3	53·6	14·7	27·8	..	16·6
55	55·80	60	50·5	74·4	43·6	64·3	51·6	76·	35·9	52·9	38·6	56·8	25·5	37·6	26·5	39·4	36·4	53·6	18·8	27·8	..	16·6
Third Period			£300·		£300·		£300·		£300·		£300·		£300·		£300·		£300·		£300·		.. £	300·

From the results here exhibited, it will be observed—

1. That according to the first three (which I will, for convenience, call first class) methods of division, the shares of each age in cash are constant—the same at one division as at another.

2. That the remaining methods—or second class—(omitting No. 6), give generally nearly similar results with one or other of the first class methods as regards the first period.

3. That at the second and third periods a rapid augmentation in the profits of the old policies takes place under the second class methods—omitting No. 7.

4. That this augmentation mainly goes to the old *lives* in No. 4 (i. and ii.); in the remaining portion of the second class methods it goes to the old *policies*, which retain very nearly the same proportion of profit, one with another, as at the first division.

5. That the bonus on policies of five years' standing rapidly diminishes with the increase in the number of old policies.

It has been observed, as regards No. 4 (i. and ii.), that "at each septennial division the same degree, or very nearly the same degree, of injustice is perpetrated as at the first"*—a little consideration of the *tendency* of the results will show that this is not so. The cash bonus of 25 at entry, taking the results of the third period under 4 (ii.) is, after 5 years, 16·8; after 10 years, 18·2; after 15 years, 20·1; which last is nearly equal to the bonus allotted at age 40 after 5 years. If this table had been worked out to a fourth period, the bonus of 25 would have been the same as 40 after 5 years. This result follows from the application of the principle as before described. The effect of this method, if continued to more distant divisions, will give, at the 11th period, to six policies taken out at the same age 25, but of different standing, the following proportions of bonus:—

	Years standing.				Years standing.
The policy of	20	would have about	$1\frac{1}{2}$	as much as policy of	5
,,	30	.. ..	2	.. ..	,,
,,	45	.. ..	4	.. ..	,,
,,	55	.. ..	8	.. ..	,,

A policy taken out at an early age, and continuing in force to the extreme age, would have successively the ratios of division for all the interval ages up to the highest.

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* *Assurance Magazine*, vol. iii., p. 191.

Of the methods not included in the table, I have but few observations to make—little more, indeed, than reasons for their omission.

From the peculiarity of the system of No. 9 (by which the annual premiums are reduced by an uniform percentage to all members, after payment of a fixed number of full premiums), no fair comparison could be instituted with the standard adopted in the table, and on this ground it was omitted. It is clear, however, that the bonuses are in proportion to the premiums, and therefore its equity mainly depends on the loading. The time of the postponement of the privilege of reduction being the same for the old as for the young lives, is also to be regarded in considering its equity. If the loading were a percentage, this rule would act very unequally, and would be unfavourable to the members entering at advanced ages.

The 10th method is neither more nor less than a pure lottery.

The 11th method, which assumes that those only are entitled to profits who have paid up their policies, is a system of distribution utterly opposed to the principles and intention of life assurance. As it would preclude profit to any policy within the period of those given in the table, it could not there be introduced.

The conditions upon which the first three methods in the table are equitable have been given. As regards the remaining methods (omitting No. 7, which I have before disposed of), if the principles laid down as to the sources of profit be borne in mind, the table of comparison establishes *that the profits contributed by the new policies are apportioned to the old, and when these new policies become old the methods provide compensation out of the contributions of the future new policies. How if these should not be forthcoming?*

I append a further table, based on Mr. Morgan's Equitable Tables. The loading in this is 15 per cent. on the pure premiums, with a constant of 5*s*. per cent. In the calculations of the bonuses according to No. 5 method, the values of the former bonuses have been included in the ratios of division. This has been done, as it is the practice of one of the Offices which has adopted this method. In other respects, the table explains itself.

Table of Comparison.

Calculations based on Mr. Morgan's Equitable Tables, 3 per cent.

Age at entry.	Premium payable.	Age at Valuation.	No. 1.		No. 2.		No. 3.		No. 4.		No. 5.	
			Cash.	Reversion.	Cash.	Reversion.	Cash.	Reversion.	Cash.	Reversion.	Cash.	Reversion.
25	21·55	30	19·5	49·4	23·9	60·5	18·5	46·9	20·2	51·15	18·6	36·8
40	32·36	45	29·2	56·7	30·6	59·3	28·9	56·	27·9	54·09	30·7	48·4
55	56·87	60	51·3	76·4	45·5	67·8	52·6	78·3	51·9	77·27	50·7	63·3
First Period			£100·		£100·		£100·		£100·		£100·	
25	21·55	35	19·5	45·2	23·9	55·4	18·5	42·9	19·8	45·9	26·9	49·4
40	32·36	50	29·2	51·6	30·6	54·1	28·9	51·1	28·5	50·4	44·5	64·7
55	56·87	65	51·3	70·9	45·5	62·7	52·6	72·5	62·2	86·	69·4	81·1
25	21·55	30	19·5	49·4	23·9	60·5	18·5	46·9	18·1	45·8	11·	21·7
40	32·36	45	29·2	56·7	30·6	59·3	28·9	56·	25·	48·5	18·2	28·7
55	56·87	60	51·3	76·4	45·5	67·8	52·6	78·3	46·4	69·1	30·	37·5
Second Period			£200·		£200·		£200·		£200·		£200·	
25	21·55	40	19·5	41·4	23·9	50·7	18·5	39·2	19·5	41·4	31·7	54·
40	32·36	55	29·2	47·2	30·6	49·4	28·9	46·5	30·7	49·6	52·3	70·3
55	56·87	70	51·3	66·4	45·5	58·7	52·6	67·8	74·6	96·5	77·3	85·1
25	21·55	35	19·5	45·2	23·9	55·4	18·5	42·9	17·3	40·1	18·6	34·1
40	32·36	50	29·2	51·6	30·6	54·1	28·9	51·1	25·	44·2	30·9	44·9
55	56·87	65	51·3	70·9	45·5	62·7	52·6	72·5	54·5	75·3	48·1	56·2
25	21·55	30	19·5	49·4	23·9	60·5	18·5	46·9	15·8	40·	7·6	15·
40	32·36	45	29·2	56·7	30·6	59·3	28·9	56·	21·9	42·5	12·6	19·9
55	56·87	60	51·3	76·4	45·5	67·8	52·6	78·3	40·7	60·6	20·9	26·1
Third Period			£300·		£300·		£300·		£300·		£300·	